Educ. Treat. Child. (2021) 44:169–183
https://doi.org/10.1007/s43494-021-00050-6

ORIGINAL ARTICLE

# An Investigation of the Use of Self-Regulated Strategy Development to Teach Long Division to Students with or At-Risk for Emotional Disturbance

**Mickey Losinski · Julie Thiele · Robin Parks Ennis · Ashley Shaw**

Accepted: 15 July 2021 / Published online: 3 September 2021


**Abstract** Division is an essential skill for math fluency and achievement in higher-order math skills. Using a partial quotients algorithm is one way to support a student's ability to develop both conceptual and computational knowledge of long division. We used the self-regulated strategy development (SRSD) framework to develop lesson plans and a mnemonic to guide students through the long division process. Using a multiple-baseline across participants design, SRSD instruction resulted in a functional relation between the intervention and both correct answers and rubric scores. The intervention was implemented by a practitioner with high levels of fidelity and had high levels of acceptability from both teacher and student perspectives. We present limitations and future directions in this area.

**Keywords** EBD · Academic · Behavioral · Elementary · Interventions · Mathematics

M. Losinski
Kansas State University, Birmingham, AL, USA

J. Thiele
Wichita State University, Birmingham, AL, USA

R. P. Ennis
University of Alabama at Birmingham, Birmingham, AL, USA

A. Shaw (✉)
Pittsburg State University, Pittsburg, AL, USA
e-mail: ajshaw@pittstate.edu

## Introduction

Mathematics difficulties are a persistent concern in the United States (Siegler et al., 2012). Recent assessment data have shown that students have consistently underperformed in the area of mathematics for more than a decade (National Assessment of Educational Progress [NAEP], 2019). A longitudinal study by Siegler et al. suggested that two mathematics concepts (fractions and division) were predictors of later mathematics performance (e.g., Algebra). In addition, the National Mathematics Advisory Panel (National Mathematics Advisory Panel, 2008) discussed the implications of building number sense fluency as a foundation for future math acquisition. Their recommendations include building a sense of fluency in division of whole numbers by the end of the 5th grade.

Mathematics outcomes for all students are dire, but for students with disabilities the outcomes are even more disturbing. Results from the 2019 Mathematics NAEP, for example, yielded substantial deviations in achievement levels when comparing students with disabilities to typically developing children. Of the four performance-level categories (Advanced, Proficient, Basic, and Below Basic), a mere 17% of assessed 4th graders identified with a disability achieved a level of Proficient or above whereas 45% of assessed 4th graders not identified with a disability met either Proficient or Advanced performance levels. Test results from 8$^{th}$-grade students proved even more dismal as only 9% of students with disabilities met Proficient levels compared to 37% of students without disabilities who met or exceeded a level of Proficient (NAEP, 2019).

When examining mathematics performance of students with disabilities, those identified with emotional disturbances, commonly referred to as students with emotional or behavioral disorders (EBD), are a unique population to observe. Studies not only show that, on average, students with EBD typically perform 1–2 years below grade level (Reid et al., 2004), but mathematics is a problematic academic content area for students with this disability demographic (Anderson et al., 2001). These poor outcomes may be a result of students with EBD having less access to the general education setting and their related standards (Mulcahy et al., 2014). In addition, these results may stem from a paucity of research in mathematics for students with EBD, especially those in the early grades (Losinski et al., 2019a).

Mathematics education has shifted teaching and learning theories, with a pendulum-like effect for nearly 100 years, the key focus behind this is the emphasis placed on either procedural or conceptual understanding (National Council of Teachers of Mathematics [NCTM], 2007). Recent shifts taking place in the Common Core Era revolve around the balance of procedural and conceptual understanding, recognizing the importance both have on students' success in mathematics. This is emphasized by effective teaching practice, building procedural fluency from conceptual understanding, as suggested by the NCTM (2014). Teaching with a balanced approach to procedural fluency and conceptual understanding encourages students to know how to do mathematics and why it works. The focus on computational learning starts with developing students conceptual understanding (Smith et al., 2017). Building procedural fluency from conceptual understanding begins with a focus on the concrete learning stage, using models to understand the math concept. The ways in which students manipulate those models can help them develop strategies that, when repeated and refined, develop into algorithms. This iterative process provides the time for students to develop understanding of the concepts while becoming fluent with procedures.

## Self-Regulated Strategy Development and Mathematics

Self-regulated strategy development (SRSD; Harris & Graham, 1999) is an instructional methodology that has been shown to be an effective instructional framework for students with a variety of disabilities. These findings are primarily found in the areas of writing, where SRSD is identified as an evidence-based practice (Losinski et al., 2014), and in reading (Sanders et al., 2020). Within the SRSD framework, a student's strengths and deficits are explicitly addressed through affective, behavioral, motivational, and meta-cognitive strategies (Harris & Graham, 1999). The SRSD method combines explicit instruction, self-regulation, self-monitoring, goal setting, and strategy instruction (Mason et al., 2012). Six stages of instruction make up the SRSD model: (1) develop background knowledge, (2) discuss it, (3) model it, (4) memorize it, (5) support it, and (6) independent performance of the strategy (Harris & Graham, 1999).

The use of SRSD in mathematics is an emerging area with consistently positive results. Beginning with word problems, Case et al. (1992) used SRSD to teach 5$^{th}$- and 6$^{th}$-grade students problem-solving skills with strategy instruction. Results of this investigation were positive in improving word problem calculations. Similar findings on word problems for four 3$^{rd}$- and 4$^{th}$-grade students with disabilities were also noted by Cassel and Reid (1996). Cuenca-Carlino et al. (2016) used SRSD to teach multistep algebraic equations to students at-risk for disabilities with results showing positive effects.

In addition, the authors have used SRSD to teach the addition and subtraction of fractions to students with and at-risk for disabilities in four studies. Results of the first investigation (Losinski et al., 2019b) showed positive results for 15 of 16 students with or at-risk for disabilities. Replicating this study, Ennis and Losinski (2019) found similar results for eight students with or at-risk for disabilities. Third, Losinski et al. (2021) used a portion of the SRSD strategy (FILMS) from the first two investigations to teach students at-risk for EBD. Results showed a functional relation between the FILMS strategy and increases in ability to compute addition and subtraction of fractions. Finally, using a regression discontinuity design, Losinski et al. (2021) intervened using FILMS with 16 students below benchmark in mathematics (44 in the control group). Although students in the treatment group had higher scores on the posttest, the local average treatment effect was not statistically significant. However, the overall difference in gain score suggested a large effect ($t(1, 56) = 2.59$; $p = 0.01$). Because these studies were the first to use SRSD to teach fractions computation, it is important to note all interventions were performed with high levels of treatment fidelity and participant acceptability.

### Partial Quotients Method of Long Division

While building procedural fluency within the concept of division, students should understand the concept of division as "how many groups of these are in that" (Bennett & Rule, 2005; Burns, 2003). For example, when dividing 262 by 7, the underlying concept students are considering is "how many groups of 7 are in 262," which encompasses the understanding of place value and the inverse relationship with multiplication. The traditional division algorithm requires students to follow a lengthy and precise step-by-step procedure that breaks down the dividend into individual digits to the extent that students rarely consider the actual division question (Ng, 1999; "how many of this are in that"), focusing on each place value in isolation, but rarely the whole number (Bennett & Rule, 2005). In this situation, the student never considers how many times 7 goes into 262. Rather, students typically memorize and repeat four rote steps—divide, multiply, subtract, bring down. This traditional algorithm focuses on the procedure but does not promote students' understanding of the concept of division (Bennett & Rule, 2005). As a result, students often experience minor lapses in attention to detail, which lead to incorrect quotients that can be hundreds or thousands away from the correct final answer. Common errors include students putting a number in the wrong place, bringing down a number at the wrong time, or writing the multiplier in the product position; these errors would inevitably provide an answer that was illogically incorrect and one that has little focus on the relationship between the values of 7 and 262. As such, it would be unlikely that students would notice their mistakes.

The partial products algorithm allows students to focus on the concept of division while developing a procedural algorithm (Burns, 2003). Using the same exercise as above, when implementing the partial products algorithm, students would start by asking, how many groups of 7 are in 262. The algorithm, i.e., step-by-step procedure, is intuitively implemented. Students continue to use what they know about the values and relationship between 7 and 262 and their knowledge of multiplication facts to determine the quotient. Figure 1 shows the partial quotients division algorithm, including student work, for this exercise.

The partial quotients algorithm encourages students to understand the concept of division "how many of this are in that"), while utilizing a procedure that accurately and efficiently produces the correct answer (Burns, 2003). In the above process, if a student did not recognize that 7 went into 262 twenty times, the student could have selected 10 times. With the partial quotients algorithm, students are encouraged to use multiplication facts they know, whereas in the traditional algorithm, they are required to find the largest value or they will inevitably find an incorrect answer (Bennett & Rule, 2005). This flexibility component of the partial quotients division algorithm allows a variety of student responses based on personal prerequisite knowledge of multiplication facts. The teacher can evaluate the multiple solution pathways to gain a deeper look into students' understanding of multiplication as the inverse operation of division.

The purpose of the current study is to investigate the use of SRSD to teach the partial quotients method of long division computation to students with or

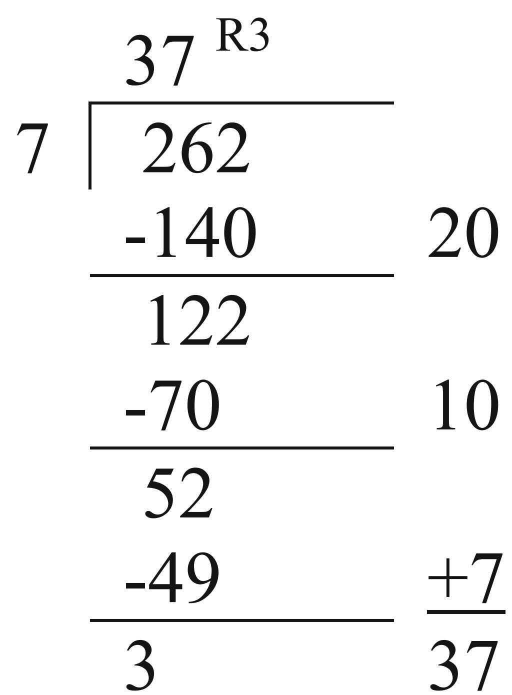


**Fig. 1** Partial Quotients Algorithm.

at-risk for EBD. This research is predicated on the following research questions: Does the SRSD partial quotients strategy improve the long division skills of students with or at-risk for EBD? and How socially valid is the SRSD partial quotients strategy for students with or at-risk for EBD?

## Method

The current study occurred in a suburban, Title I elementary school in the Midwest region of the United States. The school served nearly 500 students in kindergarten through 6$^{th}$ grades. Of the entirety of the population, 51% were male and 62% qualified for either free or reduced lunch services (13% reduced, 49% free). Students were predominantly white (60%), Hispanic (22%), or of mixed racial decent (10%). The remainder of the population was represented by an assortment of additional ethnicities. Nineteen percent of students received special education services and 11% received English as a second language support.

The school district was in its second year of implementing a three-tiered academic and behavioral system of support in which students were assessed three times per year via FastBridge, a universal, computerized assessment program (Christ, 2017) and the Social, Academic, and Emotional Behavior Risk Screener–Student and Teacher Rating Scales (SAEBRS; Kilgus et al., 2014). SAEBRS results from both students' self-ratings and teachers' ratings of internal and external social, academic, and emotional behaviors placed students into no risk, some risk, or high-risk categories. Likewise, the FASTBridge mathematics assessment placed students into one of three risk categories based upon individual student achievement. The SAEBRS screening assessment tool has adequate reliability and validity (Kilgus et al., 2016), and the FastBridge mathematics assessment is based upon recommendations from the NMAP (Christ, 2017). Results from the screeners determined which students needed intervention support at Tiers 2 and 3. Students' progress was monitored throughout the year on a bimonthly basis. The elementary school participating in this study had also just completed a review unit on multiplication and long division prior to the beginning of the study.

### Participants

#### *Inclusion and Exclusion Criteria*

Fifth-grade students were invited to participate in this study if they (1) were at-risk for mathematics on the school-delivered FASTBridge (Christ, 2017) universal screening measure in the mathematics domain; (2) were receiving services for an emotional disturbance or meeting high-risk status on the student or teacher version of the universal SAEBRS (Kilgus et al., 2014; von der Embse et al., 2017) universal screening measure for social or emotional deficits; and (3) received a failing grade on a division calculation assessment that consisted of 12 long division problems with 1-digit divisors and 3-digit dividends. Universal screening was conducted by the school district three times during the school year, in fall, winter, and spring. A sample of students was first selected according to the first two inclusion criteria. Those students were then given consent forms for parents to sign and return in order to allow students' participation in the study.

#### *Sampling Procedure*

The parents of 16 students were contacted to participate in the study, with 8 returning consent forms. The eight students who gave assent were then administered a long division assessment, with the three students performing the lowest on the assessment enrolled in the current study. All data for the study were collected daily by the intervention agent at the school site during the tiered instruction blocks in which the intervention took place.

*Rihanna* Rihanna was a 10-year-old female student of American Indian descent. She was receiving special education and related services due to academic and behavioral needs, including physical therapy and school social work services, under the Other Health Impairment (OHI) exceptionality category. In addition, Rihanna was a child served by the state's foster care system and was receiving free and reduced lunch services. She was rated by her classroom teacher as being high-risk on each of the three factors (social, emotional/behavioral/academic) of the district's SAEBRS screener.

*Tim* Tim was also a 10-year-old, twice exceptional white male receiving special education services for emotional disturbance and a specific learning disability.

He received school social work services and qualified for the district's free and reduced lunch program. Tim and his classroom teacher rated him as being at-risk on the behavioral and emotional categories of the SAEBRS screener. His classroom teacher also ranked him as at-risk in the social category of the SAEBRS assessment.

*Kevin* Kevin was a 10-year-old, twice exceptional white male receiving special education services for emotional disturbance and a specific learning disability. He was also receiving related social work services and qualified for free and reduced lunches. He was rated by both himself and his classroom teacher as being at-risk across all categories of the district's SAEBRS screener.

*Intervention agent* Intervention sessions were implemented by a 30-year-old, female special education teacher. She had 6 years teaching experience in the general education setting and was in her 3rd year of teaching in a special education resource setting. She was a 3rd-year doctoral student and had prior experience and training utilizing SRSD interventions across a variety of content areas.

## Measures

The primary outcome measure for this study was a long division probe that assessed a student's ability to calculate long division with 1-digit divisors and at least 3-digit dividends. These probes were 7-min timed assessments with 12 questions each. Probes were created using an online worksheet generator (Math-Aids.com, 2017), and randomly assigned an order of administration (i.e., baseline, intervention probes). Two methods of scoring were used in the current study; the first (primary) was a researcher-developed rubric for scoring problems, and the second was correct answers on the long division problems. The same procedures for administration and scoring were used for probes across phases: baseline, postintervention, and 2-week maintenance.

### *Rubric*

Our primary dependent variable consisted of rubric scores of completed problems. A rubric was developed that allowed the scoring to be more sensitive to change and focused on the process of arriving at an answer (Foegen et al., 2008). The rubric was scored 0–3 points per problem. Zero points were received for displaying no answer or an incorrect answer without showing work. One point was given for students arriving at no answer or an incorrect answer but correctly completing fewer than three steps in the solving of the problem. These steps consisted of the first three steps from the LSRA intervention: **L**ist easy multiples for the divisor, **S**ubtract from the dividend an easy multiple of the divisor, and **R**ecord the partial quotient to the right of the problem and repeat until the dividend is reduced to 0 or the remainder is less than the divisor. Two points were earned for arriving at no/incorrect answer, but correctly working the problem until completion (e.g., until arriving at a remainder). Three points were given for the correct answer regardless of work completed.

### *Correct Answers*

Correct answers also were calculated. To receive credit, students must have calculated the entire answer (including remainder). Correct answers were used instead of correct digits because, unlike the traditional algorithm, the partial quotients algorithm allows students to choose various partial quotients thus making it impossible to accurately count digits within the problem.

*Interrater reliability* A second researcher, a doctoral student in special education, conducted interrater reliability (IRR) on 30% of probes for both correct answers and rubric scores across all baseline, postintervention, and 2-week maintenance phases. Each rater was provided with an answer key and trained to score the probes using the above procedures. IRR was calculated by dividing the total number of agreements by the total number of opportunities. IRR for the current study was 88.9% on the rubrics and 100% on correct answers.

### *Social Validity*

Social validity was assessed by determining the acceptability of the study's goals, outcomes, and procedures (Wolf, 1978).

*Acceptability of procedures* Treatment acceptability was assessed at postintervention from the students' perspective using the Children's Intervention Rating Profile (CIRP; Witt & Elliott, 1985). The CIRP acquires social validity information from the student through a 7-item questionnaire on a 6-point Likert scale (1 = I do not agree . . . 6 = I agree) yielding a score from 7 to 42. The CIRP is a widely used measure of social validity with strong psychometric properties. Interpretation of the results of the CIRP is based on higher scores representing higher treatment acceptability. The CIRP was edited so statements used the name of the program and other specific aspects of the program while maintaining the intent of each statement.

Treatment acceptability was assessed at postintervention from the intervention agent's perspective using the Usage Rating Profile Intervention Revised (URP-IR; Chafouleas et al., 2011). The URPIR is a 29-item self-report measure designed to address the intervention agent's attitudes and feelings towards intervention acceptability*, understanding, home-school collaboration, feasibility*, system climate, and system support factors (* denotes factors having reversed-scored items). Items on the URP-IR are rated on a scale of 1 (strongly disagree) to 6 (strongly agree). As with the CIRP, the URP-IR reflects strong psychometric properties and was edited to use the name of the program and other specific aspects of the program while maintaining the intent of each statement (Chafouleas et al., 2011).

## Conditions and Design

### *Experimental Design*

The current study used a single-case, multiple-baseline across-participants design (Ledford & Gast, 2018). Multiple-baseline designs make use of longitudinal data, delivering the intervention to participants at staggered intervals to allow for the opportunity of a functional relation to be exhibited. This design was selected for several reasons, including observations from previous SRSD mathematics research (e.g., Losinski et al., 2019b), which led us to hypothesize effects on student scores would likely occur throughout intervention phases as opposed to the immediacy of noticeable results often found in other experimental designs such as pre- and posttesting. In addition, due to a small sample size, a group design would have been inappropriate for the intervention study. Test fatigue was also accounted for as it was determined a priority to start the intervention for the first participant after 3 data points were collected and then after 5 data points for participants 2 and 3 (long enough for the intervention to be delivered to each student).

### *Baseline*

Baseline performance in long division was assessed using long division probes delivered every day. Baseline probes were delivered to Rihanna, Tim, and Kevin for 3 days, before the first student (Rihanna) received the intervention. After the first student received the intervention package, baseline probes were continued for the other two students until 5 data points had been collected at which point the next student (Tim) received the intervention. This process was repeated until all of the students received the intervention.

The curriculum used by the district for 5$^{th}$ grade was *Math in Focus* (Singapore Math, Inc., 2002). All 5th-grade students, including study participants, had just completed a review of multiplication and division that included the implementation of standard calculation algorithms through general-education instruction prior to the start of this study. Study participants continued receiving general education *Math in Focus* lessons as well as their typical Tier 2 and Tier 3 mathematics instruction block during baseline and throughout all intervention phases. All tiered mathematics instruction occurred daily for 30 min and did not consist of long division lessons to ensure reliable baseline and intervention data was maintained. Typical Tier 3 behavior support (e.g., social skills training) also continued to be provided by the special education teacher to each student throughout all phases of the study as outlined in students' IEPs.

### *SRSD Mathematics*

All participants received five intervention sessions (approximately 45 min each) and were conducted individually during previously scheduled tiered intervention blocks. The intervention was comprised of self-regulated strategy development for long division. The first, third, and fourth authors developed lesson plans, and associated mnemonic devices, to teach the skills necessary to successfully complete long division equations with the partial quotients algorithm.

*Developing background knowledge* During this stage, the intervention agent discussed relevant background knowledge and vocabulary when dealing with long division. For example, during the lessons, she reviewed the terms "divisor," "dividend," and "quotient" and verified students could both identify each part of the problem and had a conceptual understanding of what each represents. Further, the intervention agent used visuals and manipulatives to demonstrate essential topics. This stage was vital in allowing the intervention agent to confirm that students had necessary preskills to use the strategy (Mason et al., 2012).

*Discussing the strategy* The second stage of SRSD involves discussing with students that they would be learning a strategy, involving a mnemonic, to help them remember all the steps of the long division algorithm (Mason et al., 2012). The intervention agent reaffirmed the benefits of learning to complete long division and was enthusiastic in learning this new approach. A learning contract was signed by the intervention agent and students to elicit joint commitment to learning and using the strategy. Once the learning contract was signed, the intervention agent modeled the steps of the algorithm using self-statements, goal setting, and self-monitoring procedures. Finally, students were presented with lesson materials (e.g., graphic organizers, mnemonic charts) and followed along as the intervention agent modeled completion of the steps.

*Modeling the strategy* In this stage, the intervention agent modeled each step of the partial quotients algorithm using "think alouds," self-instructions (self-questioning, self-praise), goal setting, and self-monitoring (Harris et al., 2008). The intervention agent led individual students in the completion of multiple problems, modeling the process of checking off each step on the strategy checklist. Students were instructed that they would complete 10 problems during the lesson and that they were going to use the strategy during each problem.

*Memorizing the strategy* Stage 4 involved actions to assist students' memorization of the mnemonic (i.e., strategy steps; Harris et al., 2008). The intervention agent began and ended most daily lessons with occasions to rehearse memorizing strategy steps through silent self-checks, partner quizzing (including with the intervention agent), or written assessment.

*Supporting the strategy* In this stage, the intervention agent aided students' practice of the strategy steps by offering opportunities for cooperative practice and scaffolding (Mason et al., 2012). For example, the intervention agent offered opportunities for students to model the steps for the intervention agent, with support, if needed.

*Independent performance* In the final stage, the intervention agent confirmed that students could use the strategy independently and without prompting (Harris et al., 2008). As students advanced through this stage, the intervention agent scaffolded supports, asking students to put away their strategy materials and, if needed, list the strategy steps at the top of the page to serve as a prompt to complete all steps.

*LSRA* The strategy used in this study was based on the partial quotients algorithm and included the mnemonic **L**ong division **S**eems **R**eally **A**wesome. The mnemonic device LSRA was developed by the first and second authors and stands for **L**ist easy multiples for the divisor, **S**ubtract from the dividend an easy multiple of the divisor, **R**ecord the partial quotient to the right of the problem and repeat until the dividend is reduced to 0 or the remainder is less than the divisor, and **A**dd the partial quotients to answer the problem. The LSRA lessons were delivered until mastery was achieved. Mastery was defined as students memorizing the LSRA strategy that accompanied the SRSD intervention (based on a quiz delivered on the 3rd day) and the ability to solve long division problems independently with correct final quotients on lesson worksheets.

*Materials* At the beginning of the LSRA strategy, students completed a learning contract where students and the interventionist committed to learning and using the strategy. Students were also given a self-instruction worksheet, mnemonic chart, checklist, and cue cards for the strategy. All students utilized every intervention material; however, the extent of use and fading of these materials depended on individual student need. For example, Tim did not rely on the mnemonic chart after the 1st day of being exposed to the mathematics tool and thus, did not require use of the chart for most of his intervention lessons. Finally, worksheets were given to students at the beginning of each intervention session to allow students to practice the targeted strategy. Each student had a folder that was passed out daily during the intervention sessions to organize these mathematics materials.

*Treatment fidelity* Treatment fidelity was assessed over 100% of sessions using a checklist of core lesson plan components by the intervention agent. Figure 2 offers an example fidelity checklist for the first intervention lesson. Due to lessons focusing on different skills, lessons ranged in steps from 13 to

**Date:___________ Instructor: _____________ Observer:______________**

**Lesson 1 Fidelity: Introduce LSRA**

**Materials**
_____ LSRA mnemonic chart, learning contract

**Reviewing Background Knowledge**
_____ Review what the students already know about long division.
_____ Review/introduce the terms partial, dividend, divisor, quotient, and multiples.

**Set the Context for Student Learning**
_____ Discuss with students the importance of understanding how to solve long division problems.
_____ Introduce partial quotients.

**Develop the Strategy and Self-Regulation**
_____ Introduce the four-step LSRA acronym.
_____ Remind students the importance of using LSRA when solving long division problems.

**Discuss the LSRA Steps**
_____ Using the mnemonic chart, explain ***L****ong division = List easy multiples of the divisor* and show examples.
_____ Explain the second letter of the acronym ***S****eems = Subtract from the dividend an easy multiple of the divisor* and show examples.
_____ Explain the third letter of the acronym ***R****eally = Record the partial quotient to the right of the problem and repeat until the dividend is reduced to zero or the remainder is less than the divisor* and show examples.
_____ Explain the final letter of the acronym ***A****wesome = Add the partial quotients to answer the problem* and show examples.

**Obtaining Commitment**
_____ Explain rational for the LSRA strategy.
_____ Explain need for student commitment.
_____ Pass out and review the learning contract content.
_____ Help students complete the learning contract.
_____ Student and teacher sign the learning contract.

**Memorization Practice**
_____ Ask students to tell you why it is important to use the LSRA strategy.
_____ Tell students it is important to memorize all the steps.
_____ Tell them they will continue to practice the strategy until they have it memorized.
_____ On a scratch piece of paper, have the students write out LSRA with a line by each letter.
_____ Review the four steps orally. As each step is stated, have students check off the blank space next to the corresponding letter.
_____ Stress again this strategy will help students when working with long division problems.

**Wrap Up**
_____ Tell students they will come to the next class and review the steps together by writing the acronym and reviewing orally.
_____ End the session with a positive praise statement.

**Total Steps Complete _____/ Total Steps Possible (24) _____=_____ *100=______**

**Fig. 2** LSRA Lesson 1 Fidelity Checklist

26 steps. Each lesson and fidelity checklist incorporated elements of the SRSD framework and its corresponding LSRA strategy; however, as students acquired stronger self-regulation skills through the framework and strategy, supports and teacher-led instruction faded. In turn, lessons required fewer items to be completed on the fidelity checklists. Items on the fidelity checklist were marked as present (1) or absent (0) by the intervention agent each day. Data on the fidelity of treatment

implementation were also collected randomly over 30% of sessions evenly spread across the intervention by another member of the research team. Fidelity of implementation was measured by dividing the total number of implemented elements by the total possible elements with the result multiplied by 100. Fidelity of implementation was 100% across all sessions for all students from the intervention agent and researcher perspective.

### Data Analysis

The primary means of data analysis was through visual analysis of graphed data (Ledford & Gast, 2018). Data were analyzed for changes in the immediacy of effect, level, variability and trend of the graphed data between baseline (A) and postintervention (B) phases. To support analysis, means, standard deviations, and slopes were calculated by phase.

To complement visual analysis, we computed the BC-SMD (Shadish et al., 2014) using the DHPS Macro (Version 1.0) for IBM SPSS (Version 23). For the analysis, we only used the baseline to postintervention data (excluding the intervention data points). Data were detrended using the session numbers as the detrending variable. The BC-SMD outcome metric has been termed one of the more robust effect measures for single-case design research and is in a metric consistent with the group standardized mean difference *g* (Shadish et al., 2014). Interpretation of the BC-SMD follows guidelines established by Cohen (1988) where BC-SMD $< 0.20$ is a small effect, $0.20 <$ BC-SMD $< 0.80$ is a moderate effect, and BC-SMD $> 0.80$ is a large effect.

## Results

### Fraction Probes

All students displayed average baseline scores of 0.00 on both outcome variables—rubric scores (Figure 3) and correct answers (Figure 4). A functional relation was established between the intervention and the outcome measures during postintervention phases. In the following, we describe data collected on the performance of individual participants after learning the SRSD division intervention (Table 1).

#### *Rihanna*

With respect to rubric scores on the division probes, Rihanna made significant gains from baseline ($M$ = 0.00, $SD$ = 0.00) to postintervention ($M$ = 4.22, $SD$ = 1.86) and continued to demonstrate skill maintenance ($M$ = 4.00, $SD$ = 1.00). With respect to correct answers, changes in level and variability were noted for Rihanna at postintervention ($M$ = 0.80, $SD$ = 0.61) with scores maintained at 2-week maintenance ($M$ = 1.00, $SD$ = 0.00).

#### *Tim*

At postintervention, the level and variability of Tim’s rubric scores improved ($M$ = 5.43, $SD$ = 1.13) with increases maintained at 2-week maintenance ($M$ = 4.33, $SD$ = 0.58). With respect to correct answers, changes in level, variability and trend were noted for Tim at postintervention ($M$ = 1.00, $SD$ = 0.82) with scores maintained at 2-week maintenance ($M$ = 1.00, $SD$ = 0.00).

#### *Kevin*

At postintervention, the level and variability of Kevin’s scores increased ($M$ = 3.25, $SD$ = 1.26) with increases maintained at 2-week maintenance ($M$ = 3.33, $SD$ = 0.58). With respect to correct answers, changes in level, variability, and trend were noted for Kevin at postintervention ($M$ = 1.00, $SD$ = 0.82) with scores maintained at 2-week maintenance ($M$ = 1.00, $SD$ = 0.00).

#### *BC-SMD*

The BC-SMD was calculated to determine the social validity of the experiment and to complement visual analysis of graphed data. BC-SMD for rubric scores was 3.38 ($SE$ = 0.63) and 1.71 ($SE$ = 0.36) correct answers. For both variables, BC-SMD met the criteria of showing a large effect.

### Social Validity

The acceptability of the intervention was assessed by surveying the students and intervention agent on their views of the intervention. On the CIRP, students rated the intervention in the following

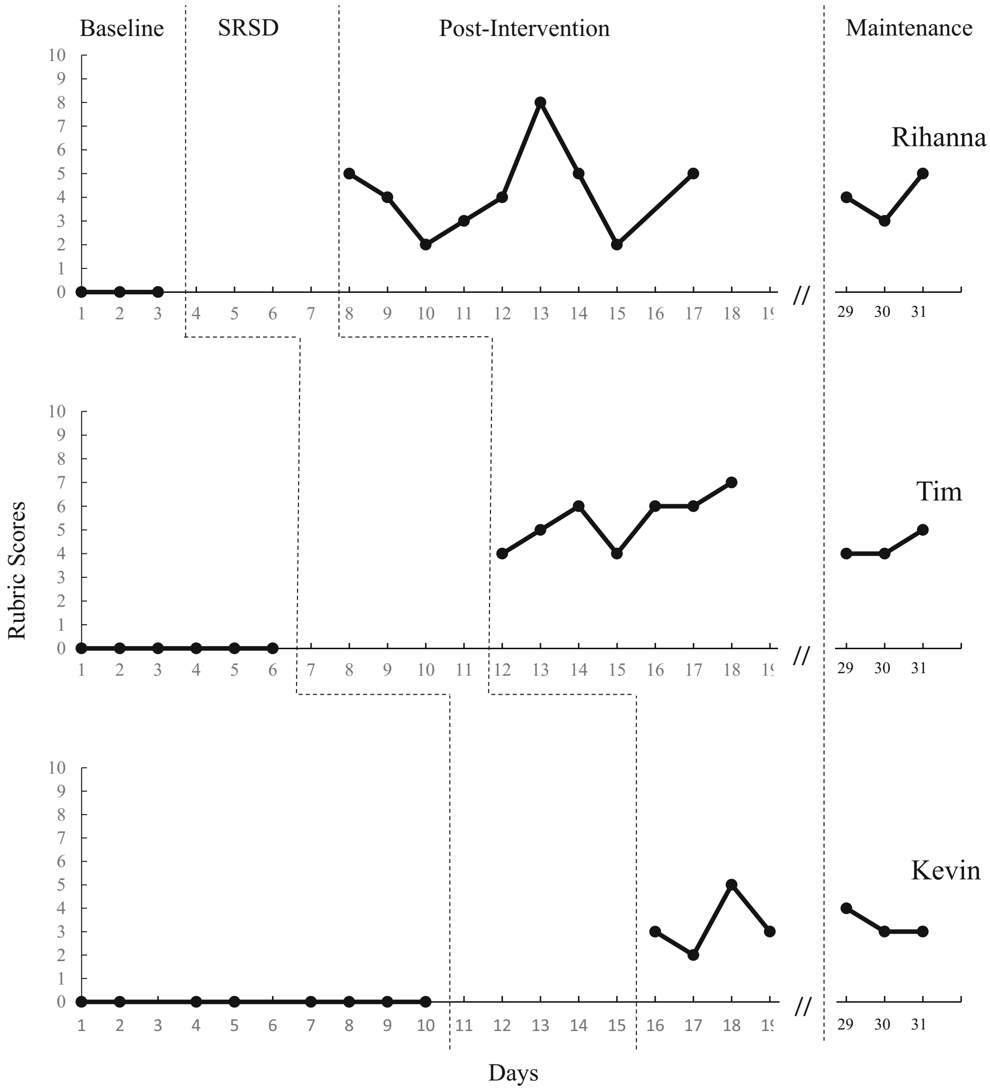


**Fig. 3** Rubric Scores

manner: Rihanna = 39, Tim = 42, Kevin = 42 (with a maximum score of 42), suggesting high levels of acceptability. On the URP-IR, the intervention agent rated items in the home collaboration (*M* = 1.66, *SD* = 0.58) and system support (*M* = 1.00, *SD* = 0.00) as low, suggesting that the intervention agent had necessary supports within the classroom and school, and thus did not require home collaboration to effectively conduct the intervention. The intervention agent rated acceptability (*M* = 4.66, *SD* = 0.87), understanding (*M* = 6.00, *SD* = 0.00), feasibility (*M* = 5.33, *SD* = 0.82), and system climate (*M* = 4.80, SD = 0.45) high with a maximum score of 6.0.

## Discussion

The current study sought to address the poor long division performance of students with and at-risk for EBD through the use of an SRSD intervention. The study used a multiple-baseline across-participants design to determine the effects of the SRSD intervention on long division probes. Results of the study showed a functional relation between the introduction of the SRSD intervention and changes in correct answers and scores on a performance rubric. In addition, the intervention was implemented with high levels of treatment fidelity and was rated as socially valid on measures of

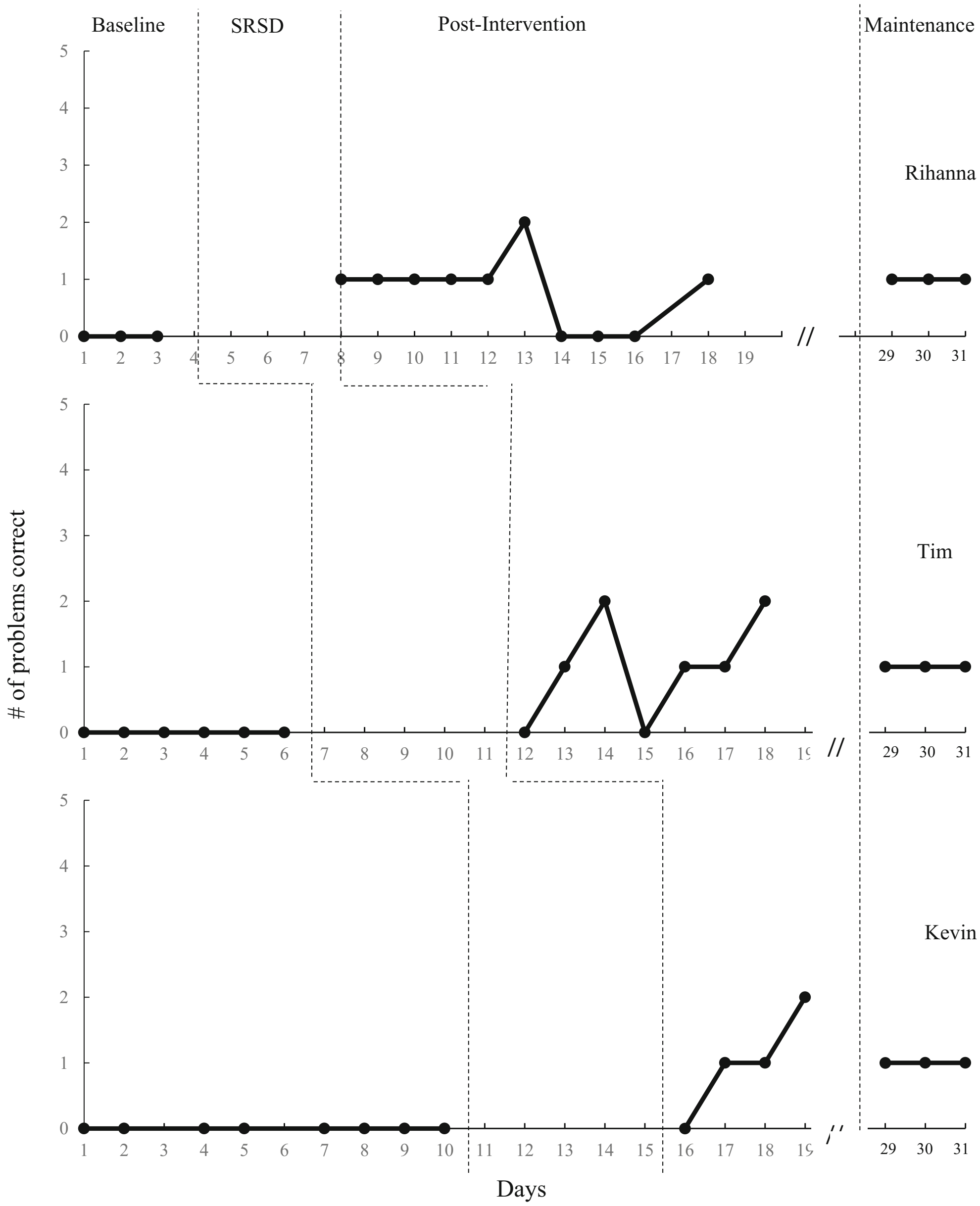


**Fig. 4** Number of Problems Correct

acceptability and effect. In the following, we discuss these results with respect to the research questions and provide limitations and directions for future research.

## Effectiveness of SRSD on Long Division

The SRSD strategy used showed a functional relation between the intervention and students' ability to complete long division items on probes. All participants improved both average rubric scores and correct answers on the division probes from baseline to postintervention and were able to demonstrate skill maintenance. In fact, all three participants were unable to accurately solve a single long division problem prior to being exposed to intervention yet were able to demonstrate a completed, accurate quotient using the SRSD strategy even after weeks of receiving the intervention. In addition, the BC-SMD of 1.71 for correct answers and 1.96 for the rubric is consistent with previous SRSD

**Table 1** Outcome Measures

| Variable | Student | Baseline | | | Postintervention | | | Maintenance (2 wk) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | *M* (*SD*) | Slope | SE | *M* (*SD*) | Slope | *SE* | *M* (*SD*) | Slope | *SE* |
| Rubric Scores | | | | | | | | | | |
| | Rihanna | 0.00 (0.00) | 0.00 | 0.00 | 4.22 (1.86) | 0.08 | 1.97 | 4.00 (1.00) | 0.50 | 1.22 |
| | Tim | 0.00 (0.00) | 0.00 | 0.00 | 5.43 (1.13) | 0.39 | 0.82 | 4.33 (0.58) | 0.50 | 0.41 |
| | Kevin | 0.00 (0.00) | 0.00 | 0.00 | 3.25 (1.26) | 0.30 | 1.47 | 3.33 (0.58) | -0.50 | 0.41 |
| Correct Problems | | | | | | | | | | |
| | Rihanna | 0.00 (0.00) | 0.00 | 0.00 | 0.80 (0.61) | -0.07 | 0.61 | 1.00 (0.00) | 0.00 | 0.00 |
| | Tim | 0.00 (0.00) | 0.00 | 0.00 | 1.00 (0.82) | 0.18 | 0.79 | 1.00 (0.00) | 0.00 | 0.00 |
| | Kevin | 0.00 (0.00) | 0.00 | 0.00 | 1.00 (0.82) | 0.60 | 0.32 | 1.00 (0.00) | 0.00 | 0.00 |

*Note:* The primary dependent variable for this study was rubric scores (0–3 points per completed problem). *M* = mean, *SD* = standard deviation, *SE* = standard error.

mathematics intervention research that showed large effects for fractions (Losinski et al., 2019b; Ennis & Losinski, 2019; Losinski et al. 2021; Losinski et al., in press), word problems (Case et al., 1992; Cassel & Reid, 1996), and Algebra (Cuenca-Carlino et al., 2016). With respect to remediating the mathematics skills of students with EBD, the current study is consistent with Losinski et al. (2019b) and provides evidence that SRSD can be an effective method for improving the mathematics skills of these students. This is likely owed to the prominence of self-regulation strategies including self-monitoring and goal setting often used with students with EBD (Mooney et al., 2005). The results of the current study are promising given the necessity for developing evidence-based academic interventions for students with or at-risk for EBD (Ennis & Jolivette, 2014).

## Social Validity

The social validity of the current study was rated as high by students and the intervention agent on the CIRP and URP-IR, respectively. This is consistent with other studies using SRSD in mathematics (e.g., Losinski et al., 2021; Cuenca-Carlino et al., 2016) that showed high levels of acceptability of procedures, goals, and outcomes. Given the large effects on students' mathematics performance, we can also conclude that SRSD intervention had acceptable outcomes. Considering this is the first study to incorporate SRSD and long division, these positive results in social validity are especially essential in planning for future replica studies.

## Limitations and Future Directions

There are a number of limitations of the current study that must be addressed. The first limitation is that students selected for the study were those with or at-risk for EBD, with one of the students being served for an Other Health Impairment (OHI), though her behaviors were observed to have the greatest impact on student learning. Therefore, it is possible that the results shown here may differ from those with students all classified as EBD. Further, the students in this study were all served in the general education classroom for a majority of the day; therefore, it is not clear if this methodology would work as well for students with more significant behavioral challenges. Future researchers should examine the impact of the SRSD framework along the continuum of placements for students with EBD.

Second, the algorithm used in the study (the partial quotients method) was different than what students were traditionally taught in schools. Thus, it is unclear if teaching students to use the traditional algorithm using SRSD components would have made a stronger impact on student performance. Future researchers should examine the relative advantages of the two methods as well as their comparative effects when implemented using the SRSD framework. Third, the probes used in this study have not been investigated for reliability and validity beyond the interrater reliability assessed for this study; thus, results should be taken with caution. Future researchers should examine the utility of using rubrics to assess student mastery of concepts and procedures in mathematics within a single-case design experiment. It should also be noted that the interventionist herself was directly involved in the scoring of the probes. Although a neutral second observer also scored the probes for reliability purposes, there is a general risk for possible bias in cases where the interventionist is directly involved in data collection and analyzation.

Fourth, test fatigue—derived from taking the probes over 20 consecutive days—may have set in for each of the students with variability in scores possibly being related to this effect. In addition, although we assessed maintenance, we waited only a relatively short amount of time for this follow-up assessment (2 weeks). Future researchers should examine the effects of SRSD math instruction at longer intervals. Furthermore, in the current study, the intervention was implemented as a part of the student's tiered math instruction one-to-one with students with or at-risk for EBD. Although this may not be an accurate depiction of the type of services offered during tiered mathematics instruction, prior research (Losinski et al., 2019b) has shown the SRSD method can be used in larger groups more reminiscent of tiered mathematics instruction.

Finally, we chose rubric scores as the primary dependent variable as the correct answers was less sensitive to change. Although a functional relation and large effect were observed, students' performance only increased slightly over baseline on both measures. One reason for this is the students had a difficult time answering more than one problem in the allotted (7 min) timeframe. This is consistent with previous research using mathematics probes in this way. For example, Losinski et al. (2019b) noted that students in their study had difficulty completing fraction problems in 2 min. Further, Ennis and Losinski (2019) used a longer time frame (4 min) but suffered from the time constraints noted previously. In the current study, we used the time frame of 7 min because that is the time suggested by Foegen et al. (2008) for algebraic progress monitoring. Although this time frame was still not conducive to allowing the students to solve more problems, we think any longer might have resulted in additional testing fatigue. During baseline, students attempted more problems, but did not use correct algorithms. For example, Rihanna multiplied the dividend by the divisor before stopping on the first step on a number of problems. Her frustration with not knowing the appropriate strategy to use was noted by her writing "help" across the page on the first probe. Probe fatigue seemed to continue during postintervention and maintenance phases as well. Rihanna, who made marked gains during intervention, began to complain about taking the probes during postintervention, in particular in the 2nd week of postintervention. This may be due to deficits in number sense and automaticity that may have slowed these students more than typical peers. This is of note because we selected the three lowest performing students (of eight assessed) for intervention. Future researchers may want to consider using the partial quotients approach with students who are more fluent with basic operations. However, results from the study showed that students with EBD can learn to effectively use the partial quotients strategy for solving long division.

## Conclusion

Findings from the current study support the continued research of SRSD in mathematics, with marked gains shown for each of the students with or at-risk for EBD. In addition, the current study used a novel method of calculating long division problems, the partial quotients method. Results showed that students were able to internalize the strategy and use it to correctly solve long division problems, however time limitations were noted. Further, high ratings of social validity and treatment fidelity suggest that this method may be easy to use and implement for classroom teachers.

### Declarations

**Conflicts of Interest** The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Research Involving Human Participants and/or Animals** All research has been conducted under the approval of the Kansas State University Committee for Research Involving Human Subjects (IRB).

**Informed Consent** Informed consent was obtained for all individual participants included in this study.

## References

Anderson, J. A., Kutash, K., & Duchnowski, A. J. (2001). A comparison of the academic progress of students with EBD and students with LD. *Journal of Emotional & Behavioral*

*Disorders, 9*(2), 106–115. https://doi.org/10.1177/106342660100900205

Bennett, P., & Rule, A. C. (2005). Hands-on long division with skittles for students with learning disabilities. *TEACHING Exceptional Children Plus, 1*(5), 2 http://escholarship.bc.edu/education/tecplus/vol1/iss5/2.

Burns, M. (2003). Marilyn Burns demystifies division. *Instructor, 112*(7), 18–19.

Case, L. P., Harris, K. R., & Graham, S. (1992). Improving the mathematical problem-solving skills of students with learning disabilities: Self-regulated strategy development. *Journal of Special Education, 26*, 1–19. https://doi.org/10.1177/002246699202600101

Cassel, J., & Reid, R. (1996). Use of a self-regulated strategy intervention to improve word problem-solving skills of students with mild disabilities. *Journal of Behavioral Education, 6*, 153–172. https://doi.org/10.1007/BF02110230

Chafouleas, S. M., Briesch, A. M. Neugebauer, S. R. & Riley-Tillman, T. C. (2011). *Usage rating profile-Intervention* (Rev. ed.). University of Connecticut.

Christ T. J. (2017). FastBridge learning. http://www.fastbridge.org

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Lawrence Erlbaum Associates.

Cuenca-Carlino, Y., Freeman-Green, S., Stephenson, G. W., & Hauth, C. (2016). Self-regulated strategy development instruction for teaching multi-step equations to middle school students struggling in mathematics. *Journal of Special Education, 50*(2), 75–85. https://doi.org/10.1177/0022466915622021

Ennis, R. P., & Jolivette, K. (2014). Existing research and future directions for self-regulated strategy development with students with and at-risk for E/BD. *Journal of Special Education, 48*, 32–45. https://doi.org/10.1177/0022466912454682

Ennis, R. P., & Losinski, M. L. (2019). Self-regulated strategy development for fractions: A replication study investigating FILMS, CUT, and EDIT with at-risk 5th graders. *Journal of Learning Disabilities, 52*, 399–412. https://doi.org/10.1177/0022219419859509

Foegen, A., Olson, J. R., & Impecoven-Lind, L. (2008). Developing progress monitoring measures for secondary mathematics: An illustration in algebra. *Assessment for Effective Intervention, 33*, 240–249. https://doi.org/10.1177/1534508407313489

Harris, K. R., & Graham, S. (1999). Programmatic intervention research: Illustrations from the evolution of self-regulated strategy development. *Learning Disability Quarterly, 22*, 251–262. https://doi.org/10.2307/1511259

Harris, K. R., Graham, S., Mason, L. H., & Friedlander, B. (2008). *Powerful writing strategies for all students*. Brookes Publishing.

Kilgus, S. P., Chafouleas, S. M., Riley-Tillman, T. C., & von der Embse, N. P. (2014). *Social, Academic, and Emotional Behavior Risk Screener (SAEBRS)*. Theodore J. Christ & Colleagues.

Kilgus, S. P., Eklund, K., Nathaniel, P., Taylor, C. N., & Sims, W. A. (2016). Psychometric defensibility of the Social, Academic, and Emotional Behavior Risk Screener (SAEBRS) teacher rating scale and multiple gating procedure within elementary and middle school samples. *Journal of School Psychology, 58*, 21–39. https://doi.org/10.1016/j.jsp.2016.07.001

Ledford, J. R., & Gast, D. L. (2018). *Single case research methodology: Applications in special education and behavioral sciences*. Routledge.

Losinski, M., Cuenca-Carlino, Y., Zablocki, M., & Teagarden, J. (2014). Examining the efficacy of self-regulated strategy development for students with emotional or behavioral disorders: A meta-analysis. *Behavioral Disorders, 40*, 52–67. https://doi.org/10.17988/0198-7429-40.1.52

Losinski, M., Ennis, R. P., Sanders, S. A., & Nelson, J. (2019a). Examining the evidence-base of mathematics interventions for students with emotional disturbances: A meta-analysis. *Journal of Special Education, 52*, 228–241. https://doi.org/10.1177/0022466918796200

Losinski, M. L., Ennis, R. P., Sanders, S., & Wiseman, N. (2019b). An investigation of self-regulated strategy development to teach addition and subtraction of fractions with unlike denominators to students with or at-risk for disabilities. *Exceptional Children, 85*, 291–308. https://doi.org/10.1177/0014402918813980

Losinski, M., Ennis, R. P., & Shaw, A. (2021). Using SRSD to improve the fraction computations of students with and at-risk for EBD. *Behavioral Disorders, 46*(2), 108–119. https://doi.org/10.1177/0198742920912737

Losinski, M., Ennis, R. P., Shaw, A., & Gage, N. (2021). Supporting students within a MTSS framework using SRSD Fractions: Results of a regression discontinuity design. *Learning Disabilities Research & Practice.*

Mason, L. H., Reid, R., & Hagaman, J. L. (2012). *Building comprehension in adolescents: Powerful strategies for improving reading and writing in content areas*. Brookes Publishing.

Math-Aids.com (2017). *Dynamically created math worksheets.* http://www.math-aids.com

Mooney, P., Ryan, J. B., Uhing, B. M., Reid, R., & Epstein, M. H. (2005). A review of self-management interventions targeting academic outcomes for students with emotional and behavioral disorders. *Journal of Behavioral Education, 14*, 203–221. https://doi.org/10.1007/s10864-005-6298-1

Mulcahy, C. C., Krezmien, M., & Maccini, P. (2014). Teaching mathematics to secondary students with emotional and behavioral disorders: Challenges and practical suggestions for teachers. *Preventing School Failure, 58*, 69–79. https://doi.org/10.1080/1045988X.2013.770727

National Assessment of Educational Progress. (2019) Mathematics assessments various years, 2005–2019. https://www.nationsreportcard.gov/mathematics/nation/achievement/

National Mathematics Advisory Panel. (2008). *Reports of the task groups and subcommittees*.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all.*

National Council of Teachers of Mathematics. (2007). *The learning of mathematics: 69th NCTM yearbook*.

Ng, S. F. (1999). Learning long division relationally: It's never too late. *Mathematics in School, 28*(4), 8–9.

Reid, R., Gonzalez, J. E., Nordness, P. D., Trout, A., & Epstein, M. H. (2004). A meta-analysis of the academic status of students with emotional/behavioral disturbance. *Journal of Special Education, 38*(3), 130–143. https://doi.org/10.1177/00224669040380030101

Sanders, S., Losinski, M., Ennis, R. P., Lane, J., White, W., & Teagarden, J. (2020). Self-regulated strategy development to improve comprehension of elementary students with and at-risk for E/BD. *Education & Treatment of Children, 43*, 21–33. https://doi.org/10.1007/s43494-020-00003-5

Shadish, W. R., Hedges, L. V., & Pustejovsky, J. E. (2014). Analysis and meta-analysis of single case designs with a standardized mean difference statistic: A primer and applications. *Journal of School Psychology, 52*, 123–147. https://doi.org/10.1016/j.jsp.2013.11.005

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23*, 691–697. https://doi.org/10.1177/0956797612440101

Singapore Math Inc. (2002). Primary mathematics scope and sequence. http://www.singaporemath.com/scope_and_sequence_PriMath.htm

Smith, M., Huinker, D. & Bill, V. (2017). *Taking action: Implementing effective mathematics teaching practices.* National Council of Teachers of Mathematics.

von der Embse, N. P., Iaccarino, S., Mankin, A., Kilgus, S. P., & Magen, E. (2017). Development and validation of the social, academic, and emotional behavior risk screener–student rating scale. *Assessment for Effective Intervention, 42*(3), 186–192. https://doi.org/10.1177/1534508416679410

Witt, J. C., & Elliott, S. N. (1985). Acceptability of classroom intervention strategies. In T. R. Kratochwill (Ed.), *Advances in school psychology (Vol. 4, pp. 251–288)*. Lawrence Erlbaum Associates.

Wolf, M. M. (1978). Social validity: The case for subjective measurement or how applied behavior analysis is finding its heart. *Journal of Applied Behavior Analysis, 11*, 203–214. https://doi.org/10.1901/jaba.1978.11-203